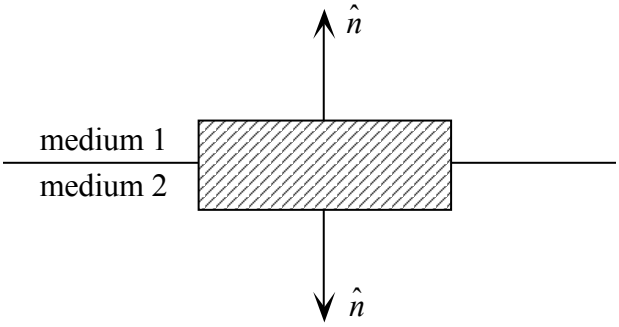
Boundary Conditions



Consider the balance of forces on a pillbox-shaped volume

body force + inertial forces + surface forces = 0

$$\int_{v} \rho \ddot{u} dv + \int_{s} \tau \cdot \hat{n}\, dA = 0$$

Let the thickness of the pillbox go to zero then

$$\int_{s} \tau \cdot \hat{n}\, dA = 0 = \int_{\substack{\text{upper} \\ \text{surface}}} \tau \cdot \hat{z}\, dA - \int_{\substack{\text{lower} \\ \text{surface}}} \tau \cdot \hat{z}\, dA = 0$$

$$\int_{\substack{\text{upper} \\ \text{surface}}} \tau \cdot \hat{z}\, dA = \int_{\substack{\text{lower} \\ \text{surface}}} \tau \cdot \hat{z}\, dA = 0$$

Now the area of the pillbox can be made arbitrarily small, hence

$$\tau_{\text{lower}} \cdot \hat{z} = \tau_{\text{upper}} \cdot \hat{z} \text{ (general interface condition)}$$

But the upper medium contains no material, hence

$$\tau_{\text{lower}} \cdot \hat{z} = \mathbf{0}, \text{ i.e. } \tau_{xz} = \tau_{yz} = \tau_{zz} = 0$$

Liquid/Solid Interface

We know $\tau_{\text{liquid}} \cdot \hat{z} = \tau_{\text{solid}} \cdot \hat{z}$ The additional conidition we impose is "anti-cavitation condition" which is $\hat{z} \cdot u_{\text{liquid}} = \hat{z} \cdot u_{\text{solid}}$, i.e. the normal components of the displacement must be continuous. Notice that all of the displacement components need not be continuous across the interface because the liquid is free to move sideways.

$$(\tau_{zz})_{\text{solid}} = (\tau_{zz})_{\text{liquid}}$$
$$(\tau_{zx})_{\text{solid}} = (\tau_{zx})_{\text{liquid}} = 0$$
$$(\tau_{zy})_{\text{solid}} = (\tau_{zy})_{\text{liquid}} = 0$$

Solid/Solid Interface

$$\boldsymbol{\tau}_1 \cdot \hat{z} = \boldsymbol{\tau}_2 \cdot \hat{z}$$
$$\boldsymbol{u}_1 \cdot \hat{z} = \boldsymbol{u}_2 \cdot \hat{z}$$

Solid/Solid (Welded Contact)

$$\boldsymbol{\tau}_1 \cdot \hat{z} = \boldsymbol{\tau}_2 \cdot \hat{z}$$
$$\boldsymbol{u}_1 = \boldsymbol{u}_2$$

All three components of displacement are continuous since the solid being in welded contact is not allowed to move sideways. In other words the vector $\left[u_1\, u_2 u_3\, \tau_{13}\tau_{23}\tau_{33}\right]^T$ is continuous.