A statistical framework for Monte Carlo mine burial modeling experiments

A technical report for the ONR Mine Burial program

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Summary

The objective of this technical report is to propose a statistical framework for Monte Carlo mine burial modeling experiments. To provide a complete prediction, a probability density for mine burial needs to be expressed as a function of (1) the percentage of a mine buried and (2) the fraction of mines deployed that are buried at a specified percentage. A simple Monte Carlo experiment, where all input conditions are drawn from *a priori* uncertainty probability density functions (pdf's), will yield only the former. To extract both functionalities, uncertainty in environmental parameters must be differentiated from variability. Uncertainty describes our lack of knowledge concerning essentially deterministic factors in the mine burial process. Variability characterizes environmental influences which lead to randomness in the result irrespective of how well known or otherwise constant the environmental parameters are. Variability is what cause there to be a range in the percentage of a mine buried as a result of a single deployment; uncertainty leads to misestimation of that range, and therefore uncertainty in the number of mines buried at a given percentage. Input parameters in the Monte Carlo simulation must be drawn separately from both uncertainty and variability pdf's, effectively squaring the number of model runs. A schematic algorithm for Monte Carlo experiments is suggested.

Author's note: This brief technical report is a first draft, and is intended to spur discussion. I believe it is important that all modeling efforts within the ONR Mine Burial program be aimed at a common target that will, in the end, produce useful predictions for Navy needs. This is my proposal for what that target ought to be. Yes? No? I would greatly appreciate constructive input and commentary in advance of producing a revised draft.

Introduction

Monte Carlo methods are likely to play a prominent role in the ONR Mine Burial Program's efforts to characterize the uncertainty associated with model-based mine burial predictions. Simply put, uncertainty in model inputs can be translated into prediction uncertainty by running a model numerous times with differing inputs as chosen from an *a priori* distribution. The mean and standard deviation of the output provide estimates of the expected outcome and its uncertainty, respectively. Where the model is very complex and non-linear, Monte Carlo methods may hold the only tractable solution.

A good example of a Monte Carlo application was presented to the 2002 ONR Mine Burial workshop in La Jolla by Linwood Vincent (my apologies to Linwood for using his work as an example). Using an established (but admittedly poor) impact modeling program, and assuming reasonable estimates on uncertainty probability density functions (pdf's) for all input variables, the results of ~1000 model runs provided the basis for a frequency histogram that could be used as an estimate of the probability density, $X(b)$, as a function the percentage of the mine buried, b (or, conversely, exposed). Figure 1 displays a schematic histogram for percentage of mine buried. A number of Navy-relevant predictions can be derived directly from this type of histogram. For example, the mean and rms of the histogram pdf provide, respectively, the expected amount of burial for any one mine and the uncertainty in that prediction. Where the Navy is concerned about how many mines might be undetectable due to complete or nearly complete burial, one could make a prediction for the fraction of mines deployed that are, say, 80% buried or more ($X_{80\text{-}100}$) by simple integration of $X(b)$ from b = 80 to 100 (Figure 1).

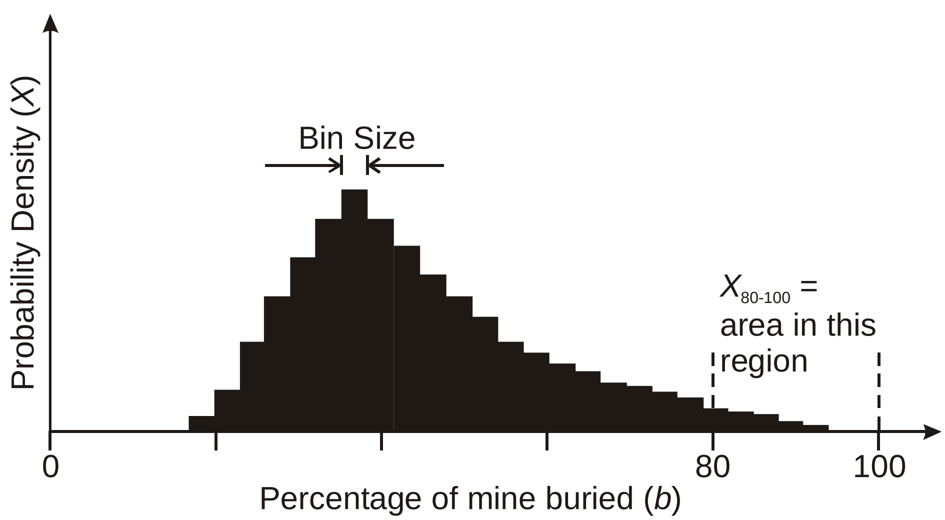

Figure 1. Schematic for a Monte Carlo-derived probability density histogram for percentage of mine buried.

It is important to bear in mind, however, that a single histogram for the percent of mine buried is only one solution among many. Linwood's example is, in one sense, a solution for the mean input conditions with respect to the *a priori* uncertainty pdf's. Provided the number of model runs is large enough, the shape of the histogram will be very stable: we will get very nearly the same histogram every time we do the Monte Carlo experiment. There will be no significant variability, for example, in the Monte Carlo prediction for $X_{80\text{-}100}$, although we know that this value should be quite uncertain if our knowledge of the deployment and burial conditions is poor. Furthermore, it is an accurate solution only if the actual mine deployment and/or burial process operated under same variability in input conditions. This is unlikely to be the case. Rather, during mine burial there will be a number of conditions that are constant or nearly so, and if they are unknown or poorly constrained, they will result in a percentage burial histogram that is different from that derived by the "mean condition" Monte Carlo run.

The basic problem with the simple Monte Carlo experiment is that it yields a one dimensional histogram when two dimensions are required: variability in the mine burial process must be expressed relative to both the percentage of mine buried and the numbers of mines buried at a specified percentage.

The Monte Carlo Algorithm

To provide a complete prediction, Monte Carlo mine burial experiments must separately account for uncertainty and variability in input parameters. This distinction is critical: variability is what acts upon the mine burial process to create random results even with complete knowledge of the conditions; it is the unavoidably random component of the mine burial process. Perhaps a more practicable better way to consider variability is that it is what cause there to be a range in the percentage of mine burial from a set of mines that were deployed under the same conditions. Uncertainty, on the other hand, leads to misestimation of that range, and therefore uncertainty in the number of mines buried at a given percentage. An effort must therefore be made in the mine burial modeling work to establish which input parameters contribute to uncertainty, which contribute to variability, and to formulate a priori pdf's for each. For example, during an actual mine deployment, drop height may be constant but unknown a priori, whereas entry orientation might be variable. If true, then an uncertainty pdf could be constructed for drop height which expresses the range of possibilities for this input parameter, and a variability pdf could be constructed for entry orientation that expresses the likely range of entry positions during a deployment. Some input parameters may have both uncertainty and variability components. Grain size, for example, could have an uncertainty pdf associated with how well we know the average grain size over the deployment area, and a variability pdf associated with spatial variability in grain size over the same area.

Monte Carlo mine burial experiments must then draw separately from uncertainty and variability pdf's, effectively squaring the number of individual model runs that must be performed (that is, a Monte Carlo set of Monte Carlo experiments). For example, the following algorithm could be used to establish the mean and rms prediction for $X_{80\text{-}100}$: (1) choose one set of input parameters from the uncertainty pdf's; (2) run a Monte Carlo experiment, choosing input parameters from the variability pdf, to establish a histogram for percentage burial $X(b)$, and from that derive a single value for $X_{80\text{-}100}$ (Figure 1), (3) return to step (1), at each iteration building up a histogram for $X_{80\text{-}100}$; and (4), once the histogram is robustly established, derive from it the mean and rms for $X_{80\text{-}100}$.

The above algorithm can be generalized (Figure 2) to formulate a series of Monte Carlo-derived histograms which can be combined as necessary to estimate pdf's that can provide desired Navy-relevant predictions with uncertainties. Arbitrarily choosing, for demonstration purposes, 10% as a bin size for the percentage of mine buried, and Δx as the bin size for the fraction of mines that are X% buried, we define

$$B_{10p}(x)\Delta x;\ p \in (0,1,...,9);\ 0 \le x \le 1$$

$$\sum_{i=0}^{N-1} B_{10p}(i\Delta x)\Delta x = 1;\ \ N\Delta x = 1$$

as the probability density that x of the deployed mines will be $10p$% to $10(p+1)$% buried. The $B_{10p}(x)$ functions are estimated by computing histograms for X_{10p}, defined as the integration the $X(b)$ histogram from $10p$ to $10(p+1)$. As above, each $X(b)$ is derived by a Monte Carlo experiment with a constant set of input parameters drawn from the uncertainty pdf's, and variable input parameters drawn from the variability pdf's. This Monte Carlo experiment is repeated numerous times with a new set of input parameters drawn from the uncertainty pdf's, formulating a new $X(b)$ each iteration, until the $B_{10p}(x)$ histograms are well resolved (Figure 2).

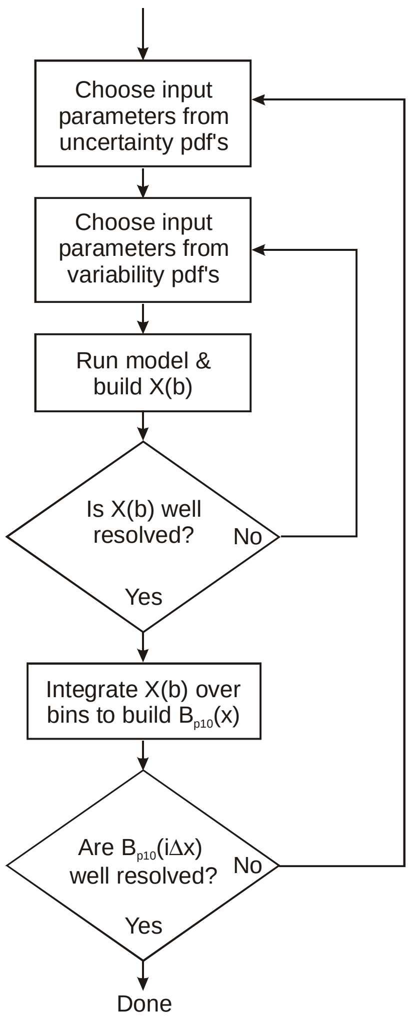

Figure 2. Schematic algorithm for a Monte Carlo set of Monte Carlo simulations, drawing from uncertainty and variability pdf's, to generate the $B_{10p}(x)$ set of histograms.

The mean of each $B_{10p}(x)$ histogram provides, respectively, the expected fraction of deployed mines that are $10p$% to $10(p+1)$% buried, and the rms provides an estimate of the uncertainty. Summing the X_{10p} values can provide estimates over a larger range of percentage burial. For example, by noting that $X_{80\text{-}100} = X_{80} + X_{90}$, the probability density histogram for $X_{80\text{-}100}$ can be estimated by convolution of the B_{80} and B_{90} histograms (assuming, as seems reasonable, that X_{80} and X_{90} are weakly covariant).